

## Errata to “Formations of Monoids, Congruences, and Formal Languages”

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In the proof of [1, Proposition 5], for the case of subdirect products, the set  $B$  does not necessarily generate the monoid  $M$ . Instead, take a set  $B$  for which there exists a surjective monoid epimorphism from  $B^*$  to  $M$ .

In the proof of [1, Proposition 6], for a set  $A$ , the condition of  $\text{coEq}(A^*/C)$  being included in  $\mathcal{F}(A)$  does not necessarily implies that  $C$  is a congruence in  $\mathbb{F}(A)$ . Let us recall that for a congruence  $C$  on a free monoid  $A^*$ , the following equation holds:

$$C = \bigcap \{ \text{Eq}\langle L \rangle \mid L \in \text{coEq}(A^*/C) \}.$$

In view of this result, and in order to recover the above argument, one has to impose closure under arbitrary intersections in the definition of formation of congruences. Being closed under arbitrary intersections was not an initial requirement on the definition of formation of congruences. Therefore, we cannot move from arbitrary formations of languages to arbitrary formations of congruences. In order to solve this problem and obtain an Eilenberg-like theorem, we need to restrict ourselves to finite-index congruences and regular languages. Furthermore, one has to reconsider the notion of formation of languages by replacing items (i) and (ii), in the original definition [1, Definition 9], with the following item.

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- (i') if  $L$  and  $L'$  are languages in  $\mathcal{F}(A)$ , then  $\text{coEq}(A^*/(\text{Eq}\langle L \rangle) \cap (\text{Eq}\langle L' \rangle))$  is included in  $\mathcal{F}(A)$ .

As a consequence, the rest of the proofs involving languages only hold when restricted to the finitary case, i.e., regular languages, finite-index congruences, and finite monoids. The Eilenberg-like theorem for monoids and congruences ([1, Theorem 6]) still holds for the non-finitary case.

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[http://www.info.uaic.ro/bin/download/Annals/XXV2/XXV2\\_0corr.pdf](http://www.info.uaic.ro/bin/download/Annals/XXV2/XXV2_0corr.pdf).

## References

- [1] A. Ballester-Bolinches, E. Cosme-Llópez, R. Esteban Romero, and J.J.M.M. Rutten. Formations of monoids, congruences, and formal languages. *Scientific Annals of Computer Science*, 25(2):171–209, 2015. doi:10.7561/SACS.2015.2.171.